Σ_1^1-computability and realisability of a global well-ordering

(Abstract only)

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Using Kleene's normal form theorem for Σ_1^1-formulae, we can organise the collection of Σ_1^1-definable partial functions into a partial combinatory algebra. This can then be used as a realisability structure to interpret intuitionistic theories of arithmetic that allow comprehension over all arithmetic formulae.

In this talk, I will use CM, a novel formal system of intuitionistic third-order arithmetic with a classical first-order fragment in Weaver [1], as an example to discuss the increase in power of this realisability model as we move to stronger fragments of second-order arithmetic in the meta-theory. The goal is to interpret an axiom of global well-ordering on all second-order objects, which functions as a choice-like extension of CM and provides an analogue of Zorn's lemma for the theory.

We will demonstrate that this approach gives an ordinal analysis of Weaver's theory. CM itself will have the proof-theoretic ordinal $\varphi_{\varepsilon_0}0$, while the addition of the global well-ordering will increase its strength to the Bachmann–Howard ordinal.

References

[1] Nik Weaver, *Axiomatizing mathematical conceptualism in third order arithmetic* (2009), available at `arXiv:0905.1675[math.HO]`.