▸ SHUWEI WANG, *The global well-ordering on Weaver's third-order conceptual mathematics.*

School of Mathematics, University of Leeds, Leeds LS2 9JT, UK.
E-mail: mmsw@leeds.ac.uk.

Around 2010, Nik Weaver proposed quite a few novel candidates for the formal foundation of mainstream mathematics, motivated by the philosophical stance he calls *mathematical conceptualism*, effectively a revival of the predicativism trend that advocates for founding mathematics upon only a small portion of the set hierarchy that is "built up from below", seeing the rest as ill-defined due to the use of impredicative definitions.

This talk will specifically be concerned with Weaver's formal system CM in [1] of intuitionistic third-order arithmetic with a classical first-order fragment, characterised by a comprehension scheme for all decidable formulae and second-order dependent choice. In the same paper, Weaver also suggested an extension of the theory by adding a primitive global well-ordering on all second-order objects. However, while he gave an abundance of examples on how various traditional topics in mathematics, such as topology, functional analysis and measure theory, can be formalised in CM, the implications of the proposed extension remain unexplored.

In this talk, I will give a proof-theoretic analysis of Weaver's theory, as well as the effect of including the extra axioms for the global well-ordering. This should bound Weaver's conceptualism between the strength of the base theory CM (with a proof-theoretic ordinal of $\varphi_{\varepsilon_0}0$) and the impredicative extension with a full transfinite induction axiom on the global well-ordering (whose strength is at the Bachmann–Howard ordinal). Interestingly, Weaver did not make clear where exactly the distinction between predicativism and impredicativism lies (though in a different paper [2] he did argue that his philosophy can justify certain theories with a proof-theoretic ordinal above Γ_0), and I will attempt to interpret what Weaver might accept as a predicative fragment of the full-strength theory.

I will additionally continue Weaver's discussion of mainstream mathematics, by demonstrating that sufficient transfinite induction on the global well-ordering can play the role of the axiom of choice for mathematics formalised in CM. In the extended theory, one can prove a restricted analogue of Zorn's lemma, from which many classical results can be derived, including the existence of a basis or the Hahn–Banach theorem for any Banach space over a complete separable $\mathbb{R}$-normed field.

[1] Nik Weaver, *Axiomatizing mathematical conceptualism in third order arithmetic*, 2009. Preprint available at arXiv:0905.1675 [math.HO].

[2] Nik Weaver, *Predicativity beyond* Γ_0, 2009. Preprint available at arXiv:math/0509244 [math.LO].