▸ SHUWEI WANG, *Realisability semantics and choice principles for Weaver's third-order conceptual mathematics.*

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In [1], Nik Weaver introduced a formal semi-intuitionistic third-order arithmetic as an alternative foundation for mainstream mathematics. This system is claimed to follow his philosophical vision of *mathematical conceptualism*, which is a variant of predicativitism and admits all and only countable procedurals built up from below as concrete mathematical objects. Supplemented by a layer of class-like third-order objects, Weaver maintains that his arithmetic can be utilised for much of modern mathematics that mainly concerns uncountable structure with a tame, countable description, such as separable metric spaces and second-countable topological spaces.

In section 2.3 of his paper, Weaver proposed a direction to further extend this theory to accommodate for uncountable choice principles that are commonly used in mainstream mathematics, by assuming a global well-ordering structure on the second-order objects. However, it remains suspectable how such an extension may stay loyal to his predicative philosophical ideals. In this talk, we shall present a realisability interpretation for Weaver's mathematics through hyperarithmetic functions in a classical second-order meta-theory. By doing this, we compare the proof-theoretic strength of Weaver's base theory and possible extensions to well-known fragments of second-order arithmetic — ranging in strength from Σ^1_1-choice to Bar induction — and use this as a benchmark to discuss how well they fit into the predicativity picture.

Detailed proofs of many results in this talk are available in the speaker's arXiv preprint [2], with some more recent improvements.

[1] Nik Weaver, *Axiomatizing mathematical conceptualism in third order arithmetic*, 2009. Preprint available at arXiv:0905.1675 [math.HO].

[2] Shuwei Wang, *An ordinal analysis of CM and its extensions*, 2025. Preprint available at arXiv:2501.12631 [math.LO].