

- SHUWEI WANG, *Indexed hierarchies and internal structure in the ordinal class in intuitionistic set theories.*

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Classically, the class Ord of all ordinals (i.e. transitive sets with transitive elements) has long served as the natural indices for transfinite hierarchies such as von Neumann's V , Gödel's L or more complicated inner models. In this talk, I will present a distinctive situation in the intuitionistic context, that various hierarchies may induce different optimal indexing classes which are subclasses of Ord.

Take the following abstract recursive construction: fix a definable set-like partial order \mathcal{R} on the class Tr of all transitive sets, which is no coarser than \subseteq yet no finer than \subseteq , then the \mathcal{R} -hierarchy $H_{\mathcal{R}} : V \rightarrow \text{Tr}$ can be defined for all set indices $x \in V$ as

$$H_{\mathcal{R}}(x) = \bigcup_{y \in x} P_{\mathcal{R}}(H_{\mathcal{R}}(y)),$$

where $P_{\mathcal{R}}(x) = \{y : y \mathcal{R} x\}$ is the \mathcal{R} -predecessor set. When $\mathcal{R} = \subseteq$, $H_{\mathcal{R}}(\alpha)$ is exactly V_{α} on the von Neumann hierarchy; when \mathcal{R} admits only first-order definable subsets instead, the recursion produces Gödel's constructible universe L .

My talk focuses on the central question: how to produce a *maximal* subclass of V on which $H_{\mathcal{R}}$ is an injection? Classically, Ord is the trivial answer regardless of \mathcal{R} , but the proof fails in intuitionistic theories such as IKP or CZF. In this talk, I will present general instructions to construct such a maximal subclass $O_{\mathcal{R}} \subseteq \text{Ord}$ for any (nice enough) specific \mathcal{R} . Through realisability arguments, I show it is consistent that hierarchies like V or L yield different indexing classes that are *proper* subclasses of Ord.

The general construction is inspired by Taylor's *plump ordinals* [1], which fits the role of $O_{\mathcal{R}}$ for the V -hierarchy. I will discuss some applications of these ordinal subclasses (where they are more useful than Ord), such as covered in my paper [2].

[1] PAUL TAYLOR, *Intuitionistic sets and ordinals*, **Journal of Symbolic Logic**, vol. 61 (1996), no. 3, pp. 705–744.

[2] SHUWEI WANG, *Some notes on plump ordinals*, available as [arXiv:2601.23070](https://arxiv.org/abs/2601.23070) [[math.LO](https://arxiv.org/archive/math)], 2026.